Unit 04

ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS

Define the following terms

Algebraic Expressions

When operations of addition and subtraction are applied to algebraic terms we obtain an algebraic expression. For instance, $5x^2-3x+\frac{2}{\sqrt{x}}$ and $3xy+\frac{3}{x}(x\neq 0)$ are algebraic expressions.

Polynomials

A polynomial in the variable x is an algebraic expression of the form

$$P(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots+a_1x+a_o, a_n\neq 0, \ldots.(i)$$

Where n, the highest power of x, is a non-negative integer called the degree of the polynomial and each coefficient a_n is a real number. The coefficient a_n of the highest power of x is called the leading coefficient of the polynomial. $2x^4y^3+x^2y^2+8x$ is a polynomial in two variables x and y and has degree 7.

Rational Express Expression

The quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x)$ is a non-zero polynomial, is called a rational expression.

For example, $\frac{2x+1}{3x+8}$, $3x+8\neq 0$ is a rational expression.

In the rational expression $\frac{p(x)}{q(x)}$, $p(x)$ is called the numerator and $q(x)$ is known as the denominator of the rational expression $\frac{p(x)}{q(x)}$. The rational expression $\frac{p(x)}{q(x)}$ need not be a polynomial.

Example

Reduce the following algebraic fractions to their lowest forms.

(i) $\frac{lx+mx-ly-my}{3x^2-3y^2}$ (ii) $\frac{3x^2+18x+27}{5x^2-45}$

Solution

(i) $\frac{lx+mx-ly-my}{3x^2-3y^2}$

$$=\frac{x(l+m)-y(l+m)}{3(x^2-y^2)}$$

$$=\frac{(l+m)(x-y)}{3(x+y)(x-y)}$$

$$=\frac{l+m}{3(x+y)}$$

Which is in the lowest forms.

(ii) $$\frac{3x^2+18x+27}{5x^2-45}=\frac{3(x^2+6x+9)}{5(x^2-9)}$$

$$\frac{3(x+3)(x+3)}{5(x+3)(x-3)}$$

$$\frac{3(x+3)}{5(x-3)}$$

Which is in the lowest forms

Example

Simplify (i) $$\frac{1}{x-y}-\frac{1}{x+y}+\frac{2x}{x^2-y^2}$$

(ii) $$\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

Solution

(i) $$\frac{1}{x-y}-\frac{1}{x+y}+\frac{2x}{x^2-y^2}$$

$$=\frac{1}{x-y}-\frac{1}{x+y}+\frac{2x}{(x+y)(x-y)}$$

$$=\frac{x+y-(x-y)+2x}{(x+y)(x-y)}$$

(L.C.M of denominators)

$$=\frac{\cancel{x}+y-\cancel{x}+y+2x}{(x+y)(x-y)}$$

$$=\frac{2x+2y}{(x+y)(x-y)}$$

$$=\frac{2(x+y)}{(x+y)(x-y)}=\frac{2}{x-y}$$

(ii) $$\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

$$=\frac{2x^2}{(x^2+4)(x^2-4)}-\frac{x}{x^2-4}+\frac{1}{x+2}$$

$$=\frac{2x^2}{(x^2+4)(x+2)(x-2)}-\frac{x}{(x+2)(x-2)}+\frac{1}{x+2}$$

$$=\frac{2x^2-x(x^2+4)+(x^2+4)(x-2)}{(x^2+4)(x+2)(x-2)}=\frac{\cancel{2x^2}-\cancel{x^3}-\cancel{4x}+\cancel{x^3}+\cancel{4x}-\cancel{2x^2}-8}{(x^2+4)(x+2)(x-2)}$$

$$= \frac{-8}{(x^2+4)(x+2)(x-2)}$$

$$= \frac{-8}{(x^2+4)(x^2-4)} = \frac{-8}{x^4-16}$$

Example

Find the product $\frac{x+2}{2x-3y} \cdot \frac{4x^2-9y^2}{xy+2y}$

Solution

$$\frac{x+2}{2x-3y} \cdot \frac{4x^2-9y^2}{xy+2y} = \frac{(x+2)[(2x)^2-(3y)^2]}{(2x-3y)(x+2)y}$$

$$= \frac{\cancel{(x+2)}(2x+3y)\cancel{(2x-3y)}}{y\cancel{(x+2)}\cancel{(2x-3y)}}$$

$$= \frac{2x+3y}{y}$$

Example

Simplify $\frac{7xy}{x^2-4x+4} \div \frac{14y}{x^2-4}$

Solution

$$\frac{7xy}{x^2-4x+4} \div \frac{14y}{x^2-4}$$

$$= \frac{7xy}{x^2-4x+4} \times \frac{x^2-4}{14y}$$

$$= \frac{7xy}{(x-2)\cancel{(x-2)}} \times \frac{(x+2)\cancel{(x-2)}}{14y}$$

$$= \frac{x(x+2)}{2(x-2)}$$

Example

Evaluate $\frac{3x^2\sqrt{y}+6}{5(x+y)}$ if x = –4 and y=9

Solution

We have, by putting $x = -4$ and $y = 9$,

$$\frac{3x^2\sqrt{y}+6}{5(x+y)} = \frac{3(-4)^2\sqrt{9}+6}{5(-4+9)} = \frac{3(16)(3)+6}{5(5)} = \frac{150}{25} = 6$$

Exercise 4.1

1. Identify whether the following algebraic expression are polynomials (yes or no).

(i)	$3x^2+\frac{1}{x}-5$	No
(ii)	$3x^3-4x^2-x\sqrt{x}+3$	No
(iii)	$x^2-3x+\sqrt{2}$	Yes
(iv)	$\frac{3x}{2x-1}+8$	No

2. State whether each of the following expression is a rational expression or not.

(i)	$\frac{3\sqrt{x}}{3\sqrt{x}+5}$	No
(ii)	$\frac{x^3-2x^2+\sqrt{3}}{2+3x-x^2}$	Yes
(iii)	$\frac{x^2+6x+9}{x^2-9}$	Yes

(iv) $\dfrac{2\sqrt{x}+3}{2\sqrt{x}-3}$ No

3. Reduce the following rational expression to the lowest forms.

(i) $\dfrac{120x^2y^3z^5}{30x^3yz^2}$

$= 4x^{2-3}y^{3-1}z^{5-2}$

$= 4x^{-1}y^2z^3$

$= \dfrac{4y^2z^3}{x}$

(ii) $\dfrac{8a(x+1)}{2(x^2-1)} = \dfrac{4a(\cancel{x+1})}{(x-1)(\cancel{x+1})} = \dfrac{4a}{x-1}$

(iii) $\dfrac{(x+y)^2-4xy}{(x-y)^2} = \dfrac{x^2+y^2+2xy-4xy}{(x-y)(x-y)}$

$= \dfrac{x^2+y^2-2xy}{(x-y)(x-y)}$

$= \dfrac{(x-y)^2}{(x-y)(x-y)}$

$= \dfrac{\cancel{(x-y)^2}}{\cancel{(x-y)^2}} = 1$

(iv) $\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

$= \dfrac{\cancel{(x^3-y^3)}(x-y)^2}{\cancel{x^3-y^3}} = (x-y)^2$

(v) $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$= \dfrac{\cancel{(x+2)}(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)\cancel{(x+2)}} = \dfrac{x-1}{x-2}$

(vi) $\dfrac{x^2-4x+4}{2x^2-8} = \dfrac{(x-2)^2}{2(x^2-4)}$

$= \dfrac{(x-2)^2}{2(x-2)(x+2)}$

$= \dfrac{\cancel{(x-2)}(x-2)}{2\cancel{(x-2)}(x+2)}$

$= \dfrac{x-2}{2(x+2)}$

(vii) $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$

$= \dfrac{64x(x^4-1)}{8(x^2+1).2(x+1)}$

$= \dfrac{64x(x^4-1)}{16(x^2+1)(x+1)}$

$= \dfrac{4x(x^2+1)(x^2-1)}{(x^2+1)(x+1)}$

$= \dfrac{4x\cancel{(x^2+1)}(x-1)\cancel{(x+1)}}{\cancel{(x^2+1)}\cancel{(x+1)}}$

$= 4x(x-1)$

$\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2} = \dfrac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$

$= \dfrac{(3x+x^2-4)\cancel{(3x-x^2+4)}}{\cancel{(4+3x-x^2)}}$

$= 3x+x^2-4$

$= x^2+3x-4$

4. Evaluate (a) $\dfrac{x^3y-2z}{xz}$ **for (i) x = 3**

y = −1, z = −2.

(a) $\dfrac{(3)^3(-1)-2(-2)}{3(-2)} = \dfrac{-27+4}{-6}$

$= \dfrac{-23}{-6} = \dfrac{23}{6} = 3\dfrac{5}{6}$

(b) $\frac{x^2y^3-5z^4}{xyz}$ for x = 4, y = −2, z = −1

$$=\frac{(4)^2(-2)^3-5(-1)^4}{(4)(-2)(-1)}=\frac{-16(8)-5}{8}$$

$$=\frac{-128-5}{8}=\frac{-133}{8}=-16\frac{5}{8}$$

5. Perform the indicated operation and simplify

(i) $$\frac{15}{2x-3y}-\frac{4}{3y-2x}$$

$$=\frac{15(3y-2x)-4(2x-3y)}{(2x-3y)(3y-2x)}$$

$$=\frac{45y-30x-8x+12y}{(2x-3y)(3y-2x)}$$

$$=\frac{57y-38x}{(2x-3y)(3y-2x)}$$

$$=\frac{19\cancel{(3y-2x)}}{(2x-3y)\cancel{(3y-2x)}}=\frac{19}{2x-3y}$$

(ii) $$\frac{1+2x}{1-2x}-\frac{1-2x}{1+2x}$$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$

$$=\frac{(1+4x^2+4x)-(1+4x^2-4x)}{(1-2x)(1+2x)}$$

$$=\frac{\cancel{1}+\cancel{4x^2}+4x\cancel{-1}\cancel{-4x^2}+4x}{(1-2x)(1+2x)}$$

$$=\frac{8x}{(1-2x)(1+2x)}=\frac{8x}{1-4x^2}$$

(iii) $$\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$$

$$=\frac{(x-5)(x+5)}{(x-6)(x+6)}-\frac{x+5}{x+6}$$

$$=\frac{(x-5)(x+5)-(x+5)(x-6)}{(x+6)(x-6)}$$

$$=\frac{(x+5)[(x-5)-(x-6)]}{(x+6)(x-6)}$$

$$=\frac{(x+5)(\cancel{x}-5\cancel{-x}+6)}{(x+6)(x-6)}$$

$$=\frac{(x+5)(1)}{(x+6)(x-6)}=\frac{x+5}{x^2-36}$$

(iv) $$\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2-2xy}{(x^2-y^2)}$$

$$=\frac{(x-y)^{\cancel{2}}}{\cancel{(x-y)}(x+y)}=\frac{x-y}{x+y}$$

(v) $$\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$$

$$=\frac{x-2}{x^2+3x+3x+9}-\frac{x+2}{2(x^2-9)}$$

$$=\frac{x-2}{x(x+3)+3(x+3)}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{x-2}{(x+3)(x+3)}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{2(x-3)(x-2)-(x+3)(x+2)}{2(x-3)(x+3)(x+3)}$$

$$=\frac{2(x^2-2x-3x+6)-(x^2+2x+3x+6)}{2(x-3)(x+3)^2}$$

$$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x-3)(x+3)^2}$$

$$=\frac{2x^2-10x+12-x^2-5x-6}{2(x-3)(x+3)^2}$$

$$=\frac{x^2-15x+6}{2(x-3)(x+3)^2}$$

(vi) $$\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{x+1-(x-1)}{(x-1)(x+1)}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{\cancel{x+1}-\cancel{x+1}}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{2}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)}-\frac{4}{x^4-1}$$

$$=\frac{\cancel{2x^2}+2-\cancel{2x^2}+2}{x^4-1}-\frac{4}{x^4-1}$$

$$=\frac{4}{x^4-1}-\frac{4}{x^4-1}$$

$$=\frac{4-4}{x^4-1}$$

$$=\frac{0}{x^4-1}$$

$$=0$$

6. Perform the indicated operation and simplify:

(i) $$(x^2-49)\,\frac{5x+2}{x+7}$$

$$=(x-7)\cancel{(x+7)}\frac{5x+2}{\cancel{x+7}}$$

$$=(x-7)(5x+2)$$

(ii) $$\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}$$

$$=\frac{4(x-3)}{(x-3)(x+3)}\div\frac{2(9-x^2)}{x^2+3x+3x+9}$$

$$=\frac{4(x-3)}{(x-3)(x+3)}\div\frac{2(3-x)(3+x)}{x(x+3)+3(x+3)}$$

$$=\frac{4(x-3)}{(x-3)(x+3)}\div\frac{2(3-x)(3+x)}{(x+3)(x+3)}$$

$$=\frac{4(x-3)}{(x+3)(x-3)}\times\frac{(x+3)(x+3)}{2(3+x)(3-x)}$$

$$=\frac{2}{3-x}$$

(iii) $$\frac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$$

$$=\frac{(x^3)^2-(y^3)^2}{x^2-y^2}\div(x^4+x^2y^2+y^4)$$

$$=\frac{(x^3-y^3)(x^3+y^3)}{x^2-y^2}\div(x^4+x^2y^2+y^4)$$

$$=\frac{(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)}{x^2-y^2}$$

$$\times\frac{1}{x^4+x^2y^2+y^4}$$

$$=\frac{\cancel{(x^2-y^2)}(x^2+xy+y^2)(x^2-xy+y^2)}{\cancel{x^2-y^2}}$$

$$\times\frac{1}{x^4+x^2y^2+y^4}$$

$$=\frac{x^4+x^2y^2+y^4}{x^4+x^2y^2+y^4}=1$$

(iv) $$\frac{x^2-1}{x^2+2x+1}\cdot\frac{x+5}{1-x}$$

$$=\frac{-\cancel{(x-1)}(x+1)}{x^2+x+x+1}\cdot\frac{x+5}{(x-1)}$$

$$= \frac{-(x+1)(x+5)}{x(x+1)+1(x+1)}$$

$$= \frac{-\cancel{(x+1)}(x+5)}{(x+1)\cancel{(x+1)}} = -\frac{x+5}{x+1}$$

(v) $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

$$= \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \cdot \frac{\cancel{x}\cancel{(x+y)}}{\cancel{y}\cancel{(x+y)}} \times \frac{\cancel{y}(x-2)}{\cancel{x}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)}$$

Example

If $a+b=7$ and $a-b=3$, then find the value of (a) a^2+b^2 (b) ab

Solution

We are given that $a+b=7$ and $a-b=3$

(a) To find the value of (a^2+b^2), we use the formula

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Substituting the values $a+b=7$ and $a-b=3$, we get

$$(7)^2+(3)^2 = 2(a^2+b^2)$$

$$\Rightarrow 49+9 = 2(a^2+b^2)$$

$$\Rightarrow 58 = 2(a^2+b^2)$$

$$\Rightarrow 29 = a^2+b^2,$$

(b) To find the value of ab, we make use of the formula

$$(a+b)^2-(a-b)^2 = 4ab$$

$$\Rightarrow (7)^2-(3)^2 = 4ab,$$

$$\Rightarrow 49-9 = 4ab$$

$$\Rightarrow 40 = 4ab,$$

$$\Rightarrow 10 = ab,$$

Hence $a^2+b^2=29$ and $ab=10$.

Example

If $a^2+b^2+c^2=43$ and $ab+bc+ca=3$, then find the value of $a+b+c$.

Solution

We know that

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$$

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$\Rightarrow (a+b+c)^2 = 43+2\times3$$

(Putting $a^2+b^2+c^2=43$ and $ab+bc+ca=3$)

$$\Rightarrow (a+b+c)^2 = 49$$

$$\Rightarrow a+b+c = \pm\sqrt{49}$$

Hence $a+b+c = \pm7$

Example

If $a+b+c=6$ and $a^2+b^2+c^2=24$ then find the value of $ab+bc+ca$.

Solution

We have

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$$

$$(6)^2=24+2(ab+bc+ca)$$

$$\Rightarrow 36=24+2(ab+bc+ca)$$

$$\Rightarrow 12=2(ab+bc+ca)$$

Hence $ab+bc+ca=6$

Example

If $a+b+c=7$ and $ab+bc+ca=9$, then find the value of $a^2+b^2+c^2$.

Solution

We know that

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (7)^2 = a^2+b^2+c^2+2(9)$

$\Rightarrow 49 = a^2+b^2+c^2+18$

$\Rightarrow 31 = a^2+b^2+c^2$

Hence $a^2+b^2+c^2 = 31$

Example

If $2x-3y=10$ and $xy=2$, then find the value of $8x^3-27y^3$.

Solution

We are given that $2x-3y=10$

$\Rightarrow (2x-3y)^3 = (10)^3$

$\Rightarrow 8x^3-27y^3-3\times 2x\times 3y(2x-3y) = 1000$

$\Rightarrow 8x^3-27y^3-18xy(2x-3y) = 1000$

$\Rightarrow 8x^3-27y^3-18\times 2\times 10 = 1000$

$\Rightarrow 8x^3-27y^3-360 = 1000$

Hence

$8x^3-27y^3 = 1360$

Example

If $x+\frac{1}{x}=8$, then find the value of $x^3+\frac{1}{x^3}$

Solution

We have been given $x+\frac{1}{x}=8$

$\Rightarrow \left(x+\frac{1}{x}\right)^3 = (8)^3$

$\Rightarrow x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right) = 512$

$\Rightarrow x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = 512$

$\Rightarrow x^3+\frac{1}{x^3}+3\times 8 = 512$

$\Rightarrow x^3+\frac{1}{x^3}+24 = 512$

$\Rightarrow x^3+\frac{1}{x^3} = 512-24$

$\Rightarrow x^3+\frac{1}{x^3} = 488$

Example

If $x-\frac{1}{x}=4$, then find $x^3-\frac{1}{x^3}$

Solution

We have $x-\frac{1}{x}=4$

$\Rightarrow \left(x-\frac{1}{x}\right)^3 = (4)^3$

$\Rightarrow x^3-\frac{1}{x^3}-3x\times\frac{1}{x}\left(x-\frac{1}{x}\right) = 64$

$\Rightarrow x^3-\frac{1}{x^3}-3(4) = 64$

$\Rightarrow x^3-\frac{1}{x^3}-12 = 64$

$\Rightarrow x^3-\frac{1}{x^3} = 64+12$

$\Rightarrow x^3-\frac{1}{x^3} = 76$

Example

Factorize $64x^3+343y^3$

Solution

We have

$64x^3+343y^3 = (4x)^3+(7y)^3$

$$= (4x+7y)[(4x)^2-(4x)(7y)+(7y)^2]$$

$$= (4x+7y)(16x^2-28xy+49y^2)$$

Example

Factorize $125x^3-1331y^3$

Solution

We have

$$125x^3-1331y^3 = (5x)^3-(11y)^3$$

$$= (5x-11y)[(5x)^2+(5x)(11y)+(11y)^2]$$

$$= (5x-11y)(25x^2+55xy+121y^2)$$

Example

Factorize

$$\left(\frac{2}{3}x+\frac{3}{2x}\right)\left(\frac{4}{9}x^2-1+\frac{9}{4x^2}\right)$$

Solution

$$\left(\frac{2}{3}x+\frac{3}{2x}\right)\left(\frac{4}{9}x^2-1+\frac{9}{4x^2}\right)$$

$$=\left(\frac{2}{3}x+\frac{3}{2x}\right)\left[\left(\frac{2}{3}x\right)^2-\left(\frac{2}{3}x\right)\left(\frac{3}{2x}\right)+\left(\frac{3}{2x}\right)^2\right]$$

$$=\left(\frac{2}{3}x\right)^3+\left(\frac{3}{2x}\right)^3$$

$$=\frac{8}{27}x^3+\frac{27}{8x^3}$$

Example

Find the product $\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2+\frac{25}{16x^2}+1\right)$

Solution

$$\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16}{25}x^2+\frac{25}{16x^2}+1\right)$$

$$=\left(\frac{4}{5}x-\frac{5}{4x}\right)\left(\frac{16x^2}{25}+1+\frac{25}{16x^2}\right)$$

(rearranging)

$$=\left(\frac{4}{5}x-\frac{5}{4x}\right)\left[\left(\frac{4}{5}x\right)^2+\left(\frac{4}{5}x\right)\left(\frac{5}{4x}\right)+\left(\frac{5}{4x}\right)^2\right]$$

$$=\left(\frac{4}{5}x\right)^3-\left(\frac{5}{4x}\right)^3=\frac{64}{125}x^3-\frac{125}{64x^3}$$

Example

Find the continued product of $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

Solution

$$(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$$

$$=(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$

$$=(x^3+y^3)(x^3-y^3)=(x^3)^2-(y^3)^2=x^6-y^6$$

Exercise 4.2

1.(i) If a + b = 10 and a – b = 6 then find value of a^2+b^2.

Solution:

$$2(a^2+b^2) = (a+b)^2 + (a-b)^2$$

$$2(a^2+b^2) = (10)^2 + (6)^2$$

$$2(a^2+b^2) = 100 + 36$$

$$a^2+b^2 = \frac{136}{2} = 68$$

(ii) If a + b = 5, a – b = $\sqrt{17}$ then find value of ab.

Solution:

$$4ab = (a+b)^2 - (a-b)^2$$

$$4ab = (5)^2 - (\sqrt{17})^2$$

$$4ab = 25 - 17$$

$$4ab = 8$$

$$ab = \frac{8}{4} = 2$$

2. If $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$ find value of $ab + bc + ca$.

Solution:

$a+b+c = -1$

Squaring

$(a+b+c)^2 = (-1)^2$

$a^2+b^2+c^2+2ab+abc+2ca = 1$

$a^2+b^2+c^2+2(ab+bc+ca) = 1$

$45 + 2(ab + bc + ca) = 1$

$2(ab + bc + ca) = 1 - 45$

$2(ab + bc + (a) = -44$

$$ab + bc + ca = \frac{-44}{2} = -22$$

3. If $m+n+p = 10$, $mn + np + pm = 27$ find value of $m^2+n^2+p^2$.

Solution:

$m+n+p = 10$

Squaring both sides

$(m+n+p)^2 = (10)^2$

$m^2+n^2+p^2+2mn+2np+2mp=100$

$m^2+n^2+p^2+2(mn+np+mp)=100$

$m^2+n^2+p^2+2(27) = 100$

$m^2+n^2+p^2+54 = 100$

$m^2+n^2+p^2 = 100 - 54$

$m^2+n^2+p^2=46$

4. If $x^2 + y^2 + z^2 = 78$ and $y+yz+zx=59$ find $x + y + z$.

Solution:

$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$= x^2+y^2+z^2+2(xy+yz+zx)$

$= 78+2(59)$

$= 78 + 118$

$= 196$

$$\sqrt{(x+y+z)^{\cancel{2}}} = \sqrt{196} = \sqrt{(\pm 14)^2}$$

$x+y+z = \pm 14$

5. If $x + y + z = 12$ and $x^2 + y^2 + z^2 = 64$ find value of $xy+yz+zx$.

Solution:

$x + y + z = 12$

Squaring both sides

$(x + y + z)^2 = (12)^2$

$x^2+y^2+z^2 + 2xy+2yz+2zx = 144$

$x^2 + y^2+z^2+2(xy+yz+zx) = 144$

$64 + 2(xy+yz+zx) = 144$

$2(xy + yz+zx) = 144 - 64$

$2(xy + yz+zx) = 80$

$$xy + yz + zx = \frac{80}{2} = 40.$$

6. If $x + y = 7$ and $xy = 12$ then find value of $x^3 + y^3$.

Solution:

$x + y = 7$

$(x + y)^3 = (7)^3$

$x^3 + y^3 + 3xy(x+y) = 343$

$x^3+y^3+3(12)(7) = 343$

$x^3 + y^3 + 252 = 343$

$x^3 + y^3 = 343 - 252$

$x^3+y^3 = 91$

7. If $3x + 4y = 11$ and $xy = 12$ then find value of $27x^3 + 64y^3$.

Solution:

$3x + 4y = 11$

$(3x + 4y)^3 = (11)^3$

$(3x)^3+(4y)^3+3(3x)(4x)(3x+4y) = 1331$

$27x^3+64y^3+36xy(3x+4y) = 1331$

$27x^3+64y^3+36(12)(11) = 1331$

$27x^3+64y^3+4752 = 1331$

$27x^3+64y^3 = 1331 - 4752 = -3421$

8. If $x - y = 4$ and $xy = 21$ then find value of $x^3 - y^3$.

Solution:

$$x - y = 4$$
$$(x-y)^3 = (4)^3$$
$$x^3-y^3-3xy(x-y) = 64$$
$$x^3-y^3-3(21)(4) = 64$$
$$x^3-y^3-252 = 64$$
$$x^3 - y^3 = 64 + 252$$
$$x^3 - y^3 = 316$$

9. If $5x - 6y = 13$ and $xy = 6$ then find value of $125x^3 - 216y^3$.

Solution:

$$5x - 6y = 13$$
$$\Rightarrow (5x-6y)^3 = (13)^3$$
$$\Rightarrow (5x)^3 - (6y)^3 - 3(5x)(6y)(5x-6y) = 2197$$
$$125x^3 - 216y^3 - 90xy(5x-6y) = 2197$$
$$125x^3 - 216y^3 - 90(6)(13) = 2197$$
$$125x^3 - 216y^3 - 7020 = 2197$$
$$125x^3 - 216y^3 = 2197 + 7020$$
$$125x^3 - 216y^3 = 9217$$

10. If $x + \frac{1}{x} = 3$ then find $x^3 + \frac{1}{x^3}$.

$x + \frac{1}{x} = 3$ Cubing both sides

$$\left(x + \frac{1}{x}\right)^3 = (3)^3$$
$$x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 27$$
$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$$
$$x^3 + \frac{1}{x^3} + 3(3) = 27$$
$$x^3 + \frac{1}{x^3} = 27 - 9$$
$$x^3 + \frac{1}{x^3} = 18$$

11. If $x - \frac{1}{x} = 7$, then find value of $x^3 - \frac{1}{x^3}$

$x - \frac{1}{x} = 7$ Taking cube of both sides

$$\left(x - \frac{1}{x}\right)^3 = (7)^3$$
$$x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 343$$
$$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 343$$
$$x^3 - \frac{1}{x^3} - 3(7) = 343$$
$$x^3 - \frac{1}{x^3} - 21 = 343$$
$$x^3 - \frac{1}{x^3} = 343 + 21$$
$$x^3 - \frac{1}{x^3} = 364$$

12. If $3x + \frac{1}{3x} = 5$, then find value of $27x^3 + \frac{1}{27x^3}$

$$\left(3x + \frac{1}{3x}\right)^3 = (5)^3$$
$$(3x)^3 + \left(\frac{1}{3x}\right)^3 + 3(3x)\left(\frac{1}{3x}\right)\left(3x + \frac{1}{3x}\right) = 125$$
$$27x^3 + \frac{1}{27x^3} + 3\left(3x + \frac{1}{3x}\right) = 125$$
$$27x^3 + \frac{1}{27x^3} + 3(5) = 125$$
$$27x^3 + \frac{1}{27x^3} + 15 = 125$$

$$27x^3 + \frac{1}{27x^3} = 125 - 15$$

$$27x^3 + \frac{1}{27x^3} = 110$$

13. If $\left(5x - \frac{1}{5x}\right) = 6$, **then find value of** $125x^3 - \frac{1}{25x^3}$.

$$\left(5x - \frac{1}{5x}\right) = 6$$

Taking cube of both sides

$$\left(5x - \frac{1}{5x}\right)^3 = (6)^3$$

$$(5x)^3 - \left(\frac{1}{5x}\right)^3 - 3(\cancel{5x})\left(\frac{1}{\cancel{5x}}\right)\left(5x - \frac{1}{5x}\right) = 216$$

$$125x^3 - \frac{1}{125x^3} - 3\left(5x - \frac{1}{5x}\right) = 216$$

$$125x^3 - \frac{1}{125x^3} - 3(6) = 216$$

$$125x^3 - \frac{1}{25x^3} - 18 = 216$$

$$125x^3 - \frac{1}{125x^3} = 216 + 18$$

$$125x^3 - \frac{1}{125x^3} = 234$$

14. Factorize (i) $x^3 - y^3 - x + y$

(i) $x^3 - y^3 - x + y$

$$= (x-y)(x^2+xy+y^2) - 1(x-y)$$

$$= (x-y)\left[x^2+xy+y^2-1\right]$$

(ii) $8x^3 - \frac{1}{27y^3}$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left(2x - \frac{1}{3y}\right)\left((2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right)$$

$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$$

15. Find products, using formulae

(i) $(x^2+y^2)(x^4-x^2y^2+y^4)$

$$= (x^2)^3 + (y^2)^3$$

$$\text{Ref} = (a+b)(a^2-ab+b^2) = a^3+b^3$$

$$= x^6 + y^6$$

(ii) $(x^3-y^3)(x^6+x^3y^3+y^6)$

$$= (x^3)^3 - (y^3)^3$$

$$\text{Ref.}(a-b)(a^2+ab+b^2) = a^3-b^3$$

$$= x^9 - y^9$$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$

$(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

$$= (x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$$

$(x^2+y^2)(x^4-x^2y^2+y^4)$

$$= (x^3-y^3)(x^3+y^3)\left[(x^2)^3+(y^2)^3\right]$$

$$= \left[(x^3)^2-(y^3)^2\right]\left(x^6+y^6\right)$$

$$= (x^6-y^6)(x^6+y^6)$$

$$= (x^6)^2 - (y^6)^2$$

$$= x^{12} - y^{12}$$

16. $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)$

$(4x^4-2x^2+1)$

$$= (2x^2-1)(4x^4+2x^2+1)(2x^2+1)$$

$(4x^4-2x^2+1)$

$$= ((2x^2)^3-(1)^3)((2x^2)^3+(1)^3)$$

$= (8x^6 - 1)(8x^6 + 1)$

$= (8x^6)^2 - (1)^2$

$= 64x^{12} - 1$

Define Surd

An irrational radical with rational radicand is called a surd.

Hence the radical $\sqrt[n]{a}$ is a surd if

(i) a is rational

(ii) the result $\sqrt[n]{a}$ is irrational.

e.g., $\sqrt{3}, \sqrt{2/5}, \sqrt[3]{7}, \sqrt[4]{10}$ are surds.

But $\sqrt{\pi}$ is not surd because is π not rational.

Note: Every surd is an irrational number but every irrational number is not surd

Example

Simplify by combining similar terms.

(i) $4\sqrt{3} - 3\sqrt{27} + 2\sqrt{75}$

(ii) $\sqrt[3]{128} - \sqrt[3]{250} + \sqrt[3]{432}$

Solution

(i) $4\sqrt{3} - 3\sqrt{27} + 2\sqrt{75}$

$= 4\sqrt{3} - 3\sqrt{9\times3} + 2\sqrt{25\times3} = 4\sqrt{3} - 3\sqrt{9}\sqrt{3} + 2\sqrt{25}\times\sqrt{3}$

$= 4\sqrt{3} - 9\sqrt{3} + 10\sqrt{3} = (4-9+10)\sqrt{3} = 5\sqrt{3}$

(ii) $\sqrt[3]{128} - \sqrt[3]{250} + \sqrt[3]{432}$

$= \sqrt[3]{64\times2} - \sqrt[3]{125\times2} + \sqrt[3]{216\times2}$

$= \sqrt[3]{(4)^3\times2} - \sqrt[3]{(5)^3\times2} + \sqrt[3]{(6)^3\times2}$

$= \sqrt[3]{(4)^3}\,\sqrt[3]{2} - \sqrt[3]{(5)^3}\,\sqrt[3]{2} + \sqrt[3]{(6)^3}\,\sqrt[3]{2}$

$= 4\sqrt[3]{2} - 5\sqrt[3]{2} + 6\sqrt[3]{2} = (4-5+6)\sqrt[3]{2} = 5\sqrt[3]{2}$

Example

Simplify and express the answer in the simplest form.

(i) $\sqrt{14}\sqrt{35}$ (ii) $\dfrac{\sqrt[6]{12}}{\sqrt{3}\,\sqrt[3]{2}}$

Solution

(i) $\sqrt{14}\sqrt{35} = \sqrt{14\times35} = \sqrt{7\times2\times7\times5} = \sqrt{(7)^2\times2\times5}$

$= \sqrt{(7)^2 \times 10} = \sqrt{(7)^2} \times \sqrt{10} = 7\sqrt{10}$

(ii) We have $\dfrac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}}$

For $\sqrt{3}$ $\sqrt[3]{2}$ the L.C.M of orders 2 and 3 is 6.

Thus $\sqrt{3} = (3)^{1/2} = (3)^{3/6} = \sqrt[6]{3^3} = \sqrt[6]{27}$

and $\sqrt[3]{2} = (2)^{1/3} = (2)^{2/6} = \sqrt[6]{(2)^2} = \sqrt[6]{4}$

Hence $\dfrac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}} = \dfrac{\sqrt[6]{12}}{\sqrt[6]{27}\sqrt[6]{4}} = \dfrac{\sqrt[6]{12}}{\sqrt[6]{108}} = \sqrt[6]{\dfrac{12}{108}} = \sqrt[6]{\dfrac{1}{9}}$

Its simplest form is

$\sqrt[6]{\left(\dfrac{1}{3}\right)^2} = \left(\dfrac{1}{3}\right)^{2/6} = \left(\dfrac{1}{3}\right)^{1/3} = \sqrt[3]{\dfrac{1}{3}}$

Exercise 4.3

1. **Express each of the following surd in the simplest form.**

(i) $\sqrt{180}$

$= \sqrt{2x2x3x3x5}$

$= 2x3\sqrt{5}$

$= 6\sqrt{5}$

(ii) $3\sqrt{162}$

$= 3\sqrt{2x3x3x3x3}$

$= 3(3x3)\sqrt{2}$

$= 27\sqrt{2}$

(iii) $\dfrac{3}{4}\sqrt[3]{128}$

$= \dfrac{3}{4}(128)^{\frac{1}{3}}$

$= \dfrac{3}{4}(2x2x2x2x2x2x2)^{\frac{1}{3}}$

$= \dfrac{3}{4}\left(2^3 x 2^3 x 2\right)^{\frac{1}{3}}$

$= \dfrac{3}{4}\left(2^3\right)^{\frac{1}{3}} \times \left(2^3\right)^{\frac{1}{3}} \times 2^{\frac{1}{3}}$

$= \dfrac{3}{4}(2)(2) \times \sqrt[3]{2}$

$= 3\sqrt[3]{2}$

(iv) $\sqrt[5]{96x^6y^7z^8}$

$= \sqrt[5]{2x2x2x2x2x3x^6y^7z^8}$

$= \left(2^5 x 3 x^5.x.y^5.y^2.z^5.z^3\right)^{\frac{1}{5}}$

$= (2^5)^{\frac{1}{5}}(3)^{\frac{1}{5}}(x^5)^{\frac{1}{5}}.x^{\frac{1}{5}}.(y^5)^{\frac{1}{5}}.(y^2)^{\frac{1}{5}}.(z^5)^{\frac{1}{5}}(z^3)^{\frac{1}{5}}$

$= 2\ 3^{\frac{1}{5}}.x.x^{\frac{1}{5}}y.y^{\frac{2}{5}}.z.z^{\frac{3}{5}}$

$= 2xyz\ 3^{\frac{1}{5}}.x^{\frac{1}{5}}.y^{\frac{2}{5}}.z^{\frac{3}{5}}$

$=2xyz\sqrt[5]{3xy^2z^3}$

2. Simplify

(i) $\frac{\sqrt{18}}{\sqrt{3}.\sqrt{2}}=\frac{\sqrt{3.3.2}}{\sqrt{3}.\sqrt{2}}=\frac{3\cancel{\sqrt{2}}}{\sqrt{3}.\cancel{\sqrt{2}}}$

$=\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\cancel{3}\sqrt{3}}{\cancel{3}}=\sqrt{3}$

(ii) $\frac{\sqrt{21}\times\sqrt{9}}{\sqrt{63}}=\frac{\sqrt{3x7}\times\sqrt{3x3}}{\sqrt{3x3x7}}$

$=\frac{\sqrt{3\times7\times3\times3}}{\sqrt{3\times3\times7}}$

$=\frac{\cancel{3}\sqrt{21}}{\cancel{3}\sqrt{7}}=\sqrt{\frac{21}{7}}$

$=\sqrt{3}$

(iii) $\sqrt[5]{243x^5y^{10}z^{15}}$

$=\left(3^5.x^5y^{10}z^{15}\right)^{\frac{1}{5}}$

$=(3^5)^{\frac{1}{5}}(x^5)^{\frac{1}{5}}(y^{10})^{\frac{1}{5}}(z^{15})^{\frac{1}{5}}$

$=3xy^2z^3$

(iv) $\frac{4}{5}\sqrt[3]{125}$

$=\frac{4}{\cancel{5}}\left(\cancel{5}^{\cancel{3}}\right)^{\frac{1}{\cancel{3}}}$

$=4$

(v) $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{3x7}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{3x7x7x3}=\left(3^2x7^2\right)^{\frac{1}{2}}$

$=(3^2)^{\frac{1}{2}}x(7^2)^{\frac{1}{2}}$

$=3\ x\ 7$

$=21$

3. Simplify by combining similar terms:

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

$=\sqrt{9x5}-3\sqrt{4x5}+4\sqrt{5}$

$=3\sqrt{5}-6\sqrt{5}+4\sqrt{5}$

$=(3-6+4)\sqrt{5}$

$=(-3+4)\sqrt{5}$

$=\sqrt{5}$

(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

$=4\sqrt{3x4}+5\sqrt{3x3x3}-3\sqrt{3x5x5}$

$+\sqrt{3x2x5x2x5}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$=(8+\cancel{15}-\cancel{15}+10)\sqrt{3}$

$=18\sqrt{3}$

(iii) $\sqrt{3}(2\sqrt{3}+3\sqrt{3})$

$=\sqrt{3}\left((2+3)\sqrt{3}\right)$

$=\sqrt{3}\left(5\sqrt{3}\right)$

$=5\sqrt{3}x\sqrt{3}$

$=5(\sqrt{3x3})$

$=5(3)$

$=15$

(iv) $2(6\sqrt{5}-3\sqrt{5})$

$=2\left((6-3)\sqrt{5}\right)$

$=2(3\sqrt{5})$

$=6\sqrt{5}$

4. Simplify:

(i) $(3+\sqrt{3})(3-\sqrt{3})$

$=(3)^2-(\sqrt{3})^2$

$$=9-3$$
$$=6$$

(ii) $\left(\sqrt{5}+\sqrt{3}\right)^2$

$$=\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2+2\sqrt{5}\sqrt{3}$$
$$=5+3+2\sqrt{15}$$
$$=8+2\sqrt{15}$$

(iii) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

$$=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$$
$$=5-3$$
$$=2$$

(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

$$=\left(\sqrt{2}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2$$
$$=2-\frac{1}{3}$$
$$=\frac{6-1}{3}=\frac{5}{3}$$

(v) $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)$ $\left(x^2+y^2\right)$

$$=\left(\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right)\left((x+y)(x^2+y^2\right)$$
$$=(x-y)(x+y)(x^2+y^2)$$
$$=(x^2-y^2)(x^2+y^2)$$
$$=(x^2)^2-(y^2)^2$$
$$=x^4-y^4$$

Define monomial surd

(i) A surd which contains a single term is called a monomial surd. e.g., $\sqrt{2},\sqrt{3}$ etc.

(ii) A surd which contains sum of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

e.g., $\sqrt{3}+\sqrt{7}$ or $\sqrt{2}+5$ $\sqrt{11}-8$ etc.

(iii) If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.

Example

Rationalize the denominator $\dfrac{58}{7-2\sqrt{5}}$

Solution

To rationalize the denominator, we multiply both the numerator and denominator by the conjugate $(7+2\sqrt{5})$ of $(7-2\sqrt{5})$, i.e.

$$\frac{58}{7-2\sqrt{5}} = \frac{58}{7-2\sqrt{5}}\times\frac{7+2\sqrt{5}}{7+2\sqrt{5}} = \frac{58(7+2\sqrt{5})}{(7)^2-(2\sqrt{5})^2}$$

$$= \frac{58(7+2\sqrt{5})}{49-20}$$; (radical is eliminated in the denominator)

$$= \frac{58(7+2\sqrt{5})}{29} = 2(7+2\sqrt{5})$$

Example

Rationalize the denominator $\dfrac{2}{\sqrt{5}+\sqrt{2}}$

Solution

Multiplying both the numerator and denominator by the conjugate $(\sqrt{5}-\sqrt{2})$ of $(\sqrt{5}+\sqrt{2})$, to get

$$\frac{2}{\sqrt{5}+\sqrt{2}} = \frac{2}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{2(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{2(\sqrt{5}-\sqrt{2})}{5-2}$$

$$= \frac{2(\sqrt{5}-\sqrt{2})}{3} = \frac{2(\sqrt{5}-\sqrt{2})}{3}$$

Example

Simplify $\dfrac{6}{2\sqrt{3}-\sqrt{6}}+\dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

Solution

First we shall rationalize the denominators and then simplify. We have

$$\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$$

$$= \frac{6}{2\sqrt{3}-\sqrt{6}}\times\frac{2\sqrt{3}+\sqrt{6}}{2\sqrt{3}+\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

$$= \frac{6(2\sqrt{3}+\sqrt{6})}{(2\sqrt{3})^2-(\sqrt{6})^2}+\frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}-\frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$$

$$= \frac{6(2\sqrt{3}+\sqrt{6})}{12-6}+\frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2}-\frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{6-2}$$

$$= \frac{12\sqrt{3}+6\sqrt{6})}{6}+\frac{\sqrt{6}\sqrt{3}-\sqrt{6}\sqrt{2})}{1}-\frac{4\sqrt{3}\sqrt{6}+4\sqrt{3}\sqrt{2}}{4}$$

$$= 2\sqrt{3}+\sqrt{6}+3\sqrt{2}-2\sqrt{3}-3\sqrt{2}-\sqrt{6} = 0$$

Example

Find rational numbers x and y such that $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = x + y\sqrt{5}$

Solution

$$\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = \frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}} = \frac{(4+3\sqrt{5})^2}{(4)^2-(3\sqrt{5})^2}$$

$$= \frac{16+24\sqrt{5}+45}{16-45} = \frac{61+24\sqrt{5}}{-29}$$

$$\Rightarrow \quad \frac{-61}{29} - \frac{24}{29}\sqrt{5} = x + y\sqrt{5} \quad \text{(given)}$$

Hence, on comparing the two sides, we get

$$x = \frac{-61}{29}, \qquad y = \frac{-24}{29}$$

Example

If $x = 3+\sqrt{8}$, then evaluate

(i) $x + \frac{1}{x}$ and (ii) $x^2 + \frac{1}{x^2}$

Solution

Since $x = 3+\sqrt{8}$, therefore,

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} = \frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2}$$

$$= \frac{3-\sqrt{8}}{9-8} = 3-\sqrt{8}$$

(i) $x + \frac{1}{x} = 3+\sqrt{8}+3-\sqrt{8} = 6$

(ii) $\left(x+\frac{1}{x}\right)^2 = 36$

or $x^2 + 2x \times \frac{1}{x} + \frac{1}{x^2} = 36$

or $x^2 + \frac{1}{x^2} = 34$

Exercise 4.4

1. Rationalize the denominator

(i) $$\frac{3}{4\sqrt{3}}=\frac{3}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{4\sqrt{3x3}}$$

$$=\frac{3\sqrt{3}}{4(3)}=\frac{\sqrt{3}}{4}$$

(ii) $$\frac{14}{\sqrt{98}}=\frac{14}{7\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{\cancel{14}\sqrt{2}}{\cancel{14}}=\sqrt{2}$$

(iii) $$\frac{6}{\sqrt{8}.\sqrt{27}}=\frac{6}{2\sqrt{2}.3\sqrt{3}}$$

$$=\frac{\cancel{6}}{\cancel{6}\sqrt{6}}$$

$$=\frac{1}{\sqrt{6}}$$

$$=\frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{\sqrt{6}}{6}$$

(iv) $$\frac{1}{3+2\sqrt{5}}=\frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$=\frac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2}=\frac{3-2\sqrt{5}}{9-20}$$

$$=\frac{3-2\sqrt{5}}{-11}$$

(v) $$\frac{15}{\sqrt{31}-4}$$

$$=\frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^2}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$=\frac{\cancel{15}\left(\sqrt{31}+4\right)}{\cancel{15}}$$

$$=\sqrt{31}+4$$

(vi) $$\frac{2}{\sqrt{5}-\sqrt{3}}=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}+\sqrt{3}\right)}{\cancel{2}}$$

$$=\sqrt{5}+\sqrt{3}$$

(vii) $$\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$=\frac{\left(\sqrt{3}\right)^2+1^2-2(1)\sqrt{3}}{2}$$

$$=\frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{2(2-\sqrt{3})}{2}$$

$$= 2-\sqrt{3}$$

(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{5-3}$$

$$= \frac{(\sqrt{5})^2+(\sqrt{3})^2+2(\sqrt{5})(\sqrt{3})}{2}$$

$$= \frac{5+3+2\sqrt{15}}{2}$$

$$= \frac{8+2\sqrt{15}}{2}$$

$$= \frac{2(4+\sqrt{15})}{2}$$

$$= 4+\sqrt{15}$$

(2) Find conjugate of $x+\sqrt{y}$ **:**

(i) $3+\sqrt{7}$

Conjugate of $3+\sqrt{7}$ is $3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Conjugate of $4-\sqrt{5}$ is $4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Conjugate of $2+\sqrt{3}$ is $2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Conjugate of $2+\sqrt{5}$ is $2-\sqrt{5}$

(v) $5+\sqrt{7}$

Conjugate of $5+\sqrt{7}$ is $5-\sqrt{7}$

(vi) $4-\sqrt{15}$

Conjugate of $4-\sqrt{15}$ is $4+\sqrt{15}$

(vii) $7-\sqrt{6}$

Conjugate of $7-\sqrt{6}$ is $7+\sqrt{6}$

(viii) $9+\sqrt{2}$

Conjugate of $9+\sqrt{2}$ is $9-\sqrt{2}$

Q.3 If $x=2-\sqrt{3}$ **find** $\frac{1}{x}$

(i)

$$x = 2-\sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{4-3}$$

$$\frac{1}{x} = 2+\sqrt{3}$$

(ii) $x=4-\sqrt{17}$ **find** $\frac{1}{x}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$\frac{1}{x} = \frac{4+\sqrt{17}}{(4)^2-(\sqrt{17})^2}$$

$$= \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+\sqrt{17}}{-1}$$

$$= -(4+\sqrt{17})$$

$$= -4-\sqrt{17}$$

(iii) **If** $x=\sqrt{3}+2$**, find** $x+\frac{1}{x}$

$$x=\sqrt{3}+2$$

$$\frac{1}{x}=\frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$\frac{1}{x}=\frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$\frac{1}{x}=\frac{\sqrt{3}-2}{3-4}$$

$$\frac{1}{x}=\frac{\sqrt{3}-2}{-1}$$

$$\frac{1}{x}=-\sqrt{3}+2=2-\sqrt{3}$$

$$x+\frac{1}{x}=\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}+2$$

$$x+\frac{1}{x}=4$$

Q4. **Simplify**

(i) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\sqrt{5}-\sqrt{3}+\sqrt{2}\sqrt{5}-\sqrt{2}\sqrt{3}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{2}\sqrt{5}-\sqrt{2}\sqrt{3}}{2}$$

$$=\frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$=\frac{\sqrt{5}-\cancel{\sqrt{3}}+\cancel{\sqrt{10}}-\sqrt{6}+\sqrt{5}+\cancel{\sqrt{3}}-\cancel{\sqrt{10}}-\sqrt{6}}{2}$$

$$=\frac{2\sqrt{5}-2\sqrt{6}}{2}$$

$$=\frac{\cancel{2}\left(\sqrt{5}-\sqrt{6}\right)}{\cancel{2}}$$

$$=\sqrt{5}-\sqrt{6}$$

(ii) $\dfrac{1}{2+\sqrt{3}}+\dfrac{2}{\sqrt{5}-\sqrt{3}}+\dfrac{1}{2+\sqrt{5}}$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{2-\sqrt{3}}{4-3}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}+\frac{2-\sqrt{5}}{4-5}$$

$$=2-\sqrt{3}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}+\frac{2-\sqrt{5}}{-1}$$

$$=\cancel{2}-\cancel{\sqrt{3}}+\sqrt{5}+\cancel{\sqrt{3}}-\cancel{2}+\sqrt{5}\;=2\sqrt{5}$$

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}-\dfrac{3}{\sqrt{5}+\sqrt{2}}$

$$=\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{5}\right)^2}+\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$= \frac{\cancel{2}\left(\sqrt{5}-\sqrt{3}\right)}{\cancel{2}} + \frac{\sqrt{3}-\sqrt{2}}{1} - \frac{\cancel{3}\left(\sqrt{5}-\sqrt{2}\right)}{\cancel{3}}$$

$$= \cancel{\sqrt{5}} - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} - \cancel{\sqrt{5}} + \cancel{\sqrt{2}}$$

$$= 0$$

Q5(i) If $x = 2+\sqrt{3}$, find value of $x - \frac{1}{x}$ and $\left(x-\frac{1}{x}\right)^2$

$$x = 2+\sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$\frac{1}{x} = 2-\sqrt{3}$$

$$x - \frac{1}{x} = 2+\sqrt{3}-\left(2-\sqrt{3}\right)$$

$$= \cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$= 2\sqrt{3}$$

$$\left(x-\frac{1}{x}\right)^2 = \left(2\sqrt{3}\right)^2$$

$$\left(x-\frac{1}{x}\right)^2 = 12$$

(ii) If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find the value of $x+\frac{1}{x}$, $x^2+\frac{1}{x}$ and $x^3+\frac{1}{x^3}$

$$x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$x = \frac{\left(\sqrt{5}-\sqrt{2}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$x = \frac{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2-2\left(\sqrt{5}\right)\left(\sqrt{2}\right)}{5-2}$$

$$x = \frac{5+2-2\sqrt{10}}{3}$$

$$x = \frac{7-2\sqrt{10}}{3}$$

$$\frac{1}{x} = \frac{3}{7-2\sqrt{10}} \times \frac{7+2\sqrt{10}}{7+2\sqrt{10}}$$

$$\frac{1}{x} = \frac{3\left(7+2\sqrt{10}\right)}{(7)^2-\left(2\sqrt{10}\right)^2}$$

$$\frac{1}{x} = \frac{3\left(7+2\sqrt{10}\right)}{49-40}$$

$$\frac{1}{x} = \frac{3\left(7+2\sqrt{10}\right)}{9}$$

$$\frac{1}{x} = \frac{7+2\sqrt{10}}{3}$$

$$x+\frac{1}{x} = \frac{7-2\sqrt{10}}{3} + \frac{7+2\sqrt{10}}{3}$$

$$= \frac{7-\cancel{2\sqrt{10}}+7+\cancel{2\sqrt{10}}}{3} = \frac{14}{3}$$

Now

$$x+\frac{1}{x} = \frac{14}{3}$$

Squaring

$$\left(x+\frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$

$$x^2+\frac{1}{x^2}+2 = \frac{196}{9}$$

$$x^2+\frac{1}{x^2} = \frac{196}{9}-2$$

$$x^2+\frac{1}{x^2} = \frac{196-18}{9} = \frac{178}{9}$$

Also

$$x^3+\frac{1}{x^3}=?$$

$$x+\frac{1}{x}=\frac{14}{3}$$

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$$x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+\cancel{3}\left(\frac{14}{\cancel{3}}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$

$$=\frac{2366}{27}$$

Q6. Determine the rational numbers a and b. If

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

Given

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=a+b\sqrt{3}$$

$$\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}+\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}=a+b\sqrt{3}$$

$$\frac{\left(\sqrt{3}\right)^2+(1)^2-2\left(\sqrt{3}\right)(1)}{3-1}+\frac{\left(\sqrt{3}\right)^2+(1)^2+2\sqrt{3}}{3-1}=a+b\sqrt{3}$$

$$\frac{3+1-2\sqrt{3}}{2}+\frac{3+1+2\sqrt{3}}{2}=a+b\sqrt{3}$$

$$\frac{4-2\sqrt{3}}{2}+\frac{4+2\sqrt{3}}{2}=a+b\sqrt{3}$$

$$\frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{2}}+\frac{\cancel{2}\left(2+\sqrt{3}\right)}{\cancel{2}}=a+b\sqrt{3}$$

$$2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}=a+b\sqrt{3}$$

$$4=a+b\sqrt{3}$$

$$\Rightarrow\ a+b\sqrt{3}=4$$

Hence on comparing the two sides, we get

$$\Rightarrow\ a=4 \text{ and } b=0$$

Exercise

Q1. If $x+\frac{1}{x}=3$ **find**

(i) $x^2+\frac{1}{x^2}$ **(ii)** $x^4+\frac{1}{x^4}$

(i)

$$x+\frac{1}{x}=3$$

$$\left(x+\frac{1}{x}\right)^2=(3)^2$$

$$x^2+\frac{1}{x^2}+2=9$$

$$x^2+\frac{1}{x^2}=9-2$$

$$x^2+\frac{1}{x^2}=7$$

(ii)

$$x^4+\frac{1}{x^4}$$

$$\left(x^2+\frac{1}{x^2}\right)^2=(7)^2$$

$$x^4+\frac{1}{x^4}+2=49$$

$$x^4+\frac{1}{x^4}=49-2$$

$$x^4+\frac{1}{x^4}=47$$

Q2. **If** $x-\frac{1}{x}=2$ **find**

(i) $x^2+\frac{1}{x^2}$ **(ii)** $x^4+\frac{1}{x^4}$

(i) $x-\frac{1}{x}=2$

Squaring

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

$$x^2+\frac{1}{x^2}-2=4$$

$$x^2+\frac{1}{x^2}=4+2$$

$$x^2+\frac{1}{x^2}=6$$

(ii) $\left(x^2+\frac{1}{x^2}\right)=(6)^2$

$$x^4+\frac{1}{x^4}+2=36$$

$$x^4+\frac{1}{x^4}=36-2$$

$$x^4+\frac{1}{x^4}=34$$

Q3. **Find value of** x^3+y^3 **and** xy **if**

$x+y=5$ **and** $x-y=3$

$$4xy=(x+y)^2-(x-y)^2$$

$$4xy=(5)^2-(3)^2$$

Now

$4xy=25-9=16$

$xy=\frac{16}{4}=4$

$$x+y=5$$

taking cube both sides

$$(x+y)^3=(5)^3$$

$$x^3+y^3+3xy(x+y)=125$$

$$x^3+y^3+3(4)(5)=125$$

$$x^3+y^3+60=125$$

$$x^3+y^3=125-60$$

$$x^3+y^3=65$$

Q4. **If** $P=2+\sqrt{3}$ **find (i)** $P+\frac{1}{P}$

(ii) $P-\frac{1}{P}$ **(iii)** $P^2+\frac{1}{P^2}$ **(iv)** $P^2-\frac{1}{P^2}$

$$P=2+\sqrt{3}$$

$$\frac{1}{P}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{P}=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

i) $P+\frac{1}{P}=2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}=4$

ii) $P-\frac{1}{P}=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}=2\sqrt{3}$

iii) $P^2+\frac{1}{P^2}=?$

$$\left(P+\frac{1}{P}\right)^2=(4)^2$$

$$P^2+\frac{1}{P^2}+2=16$$

$$P^2+\frac{1}{P^2}=16-2$$

$$P^2+\frac{1}{P^2}=14$$

iv) $P^2-\frac{1}{P^2}=?$

$$P^2-\frac{1}{P^2}=\left(P+\frac{1}{P}\right)\left(P-\frac{1}{P}\right)$$

$$=(4)\left(\sqrt{3}\right)$$

$$=8\sqrt{3}$$

Q5. If $q=\sqrt{5}+2$ Find (i) $q+\frac{1}{q}$

(ii) $q-\frac{1}{q}$ (iii) $q^2+\frac{1}{q^2}$ (iv) $q^2-\frac{1}{q^2}$

Solution: $q=\sqrt{5}+2$

$$\frac{1}{q}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$\frac{1}{q}=\frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$\frac{1}{q}=\frac{\sqrt{5}-2}{1}=\sqrt{5}-2$$

(i) $$q+\frac{1}{q}=\sqrt{5}+2+\sqrt{5}-2$$

$$=2\sqrt{5}$$

(ii) $$q-\frac{1}{q}=\sqrt{5}+2-\sqrt{5}+2$$

$$=4$$

(iii) $$q^2+\frac{1}{q^2}$$

$$\left(q+\frac{1}{q}\right)^2=\left(2\sqrt{5}\right)^2$$

$$q^2+\frac{1}{q^2}+2=20$$

$$q^2+\frac{1}{q^2}=20-2$$

$$q^2+\frac{1}{q^2}=18$$

(iv) $$q^2-\frac{1}{q^2}=\left(q+\frac{1}{q}\right)\left(q-\frac{1}{q}\right)$$

$$=\left(2\sqrt{5}\right)(4)$$

$$=8\sqrt{5}$$

Q6. Simplify

i) $$\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$$

$$=\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^{\cancel{2}}-\left(\sqrt{a^2-2}\right)^{\cancel{2}}}$$

$$=\frac{\left(\sqrt{a^2+2}\right)^{\cancel{2}}+\left(\sqrt{a^2-2}\right)^{\cancel{2}}+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{\cancel{a^2}+2-\cancel{a^2}+2}$$

$$=\frac{a^2+\cancel{2}+a^2-\cancel{2}+2\sqrt{a^4-4}}{4}$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{\cancel{2}\left(a^2+\sqrt{a^4-4}\right)}{\cancel{4}}$$

$$=\frac{a^2+\sqrt{a^4-4}}{2}$$

(ii) $$\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$$

$$=\frac{1}{a-\sqrt{a^2-x^2}}\times\frac{a+\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}$$

$$-\frac{1}{a+\sqrt{a^2-x^2}}\times\frac{a-\sqrt{a^2-x^2}}{a-\sqrt{a^2-x^2}}$$

$$=\frac{a+\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}-\frac{a-\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}$$

$$= \frac{a+\sqrt{a^2-x^2}}{\cancel{a^2}-\cancel{a^2}+x^2} - \frac{a-\sqrt{a^2-x^2}}{\cancel{a^2}-\cancel{a^2}+x^2}$$

$$= \frac{a+\sqrt{a^2-x^2}}{x^2} - \frac{a-\sqrt{a^2-x^2}}{x^2}$$

$$= \frac{\cancel{a}+\sqrt{a^2-x^2}-\cancel{a}+\sqrt{a^2-x^2}}{x^2}$$

$$= \frac{2\sqrt{a^2-x^2}}{x^2}$$

Objective

1. $4x + 3y - 2$ is an algebraic____

(a) Expression
(b) Sentence
(c) Equation
(d) In equation

2. The degree of polynomial $4x^4+2x^2y$ is _____

(a) 1 (b) 2
(c) 3 (d) 4

3. $a^3 + b^3$ is equal to_____

(a) $(a-b)(a^2+ab+b^2)$
(b) $(a+b)(a^2-ab+b^2)$
(c) $(a-b)(a^2-ab+b^2)$
(d) $(a-b)(a^2+ab-b^2)$

4. $(3+\sqrt{2})(3-\sqrt{2})$ is equal to:____

(a) 7 (b) -7
(c) -1 (d) 1

5. Conjugate of Surd $a+\sqrt{b}$ is____

(a) $-a+\sqrt{b}$ (b) $a-\sqrt{b}$
(d) $\sqrt{a}+\sqrt{b}$ (d) $\sqrt{a}-\sqrt{b}$

6. $\frac{1}{a-b} - \frac{1}{a+b}$ is equal to

(a) $\frac{2a}{a^2-b^2}$ (b) $\frac{2b}{a^2-b^2}$
(c) $\frac{-2a}{a^2-b^2}$ (d) $\frac{-2b}{a^2-b^2}$

7. $\frac{a^2-b^2}{a+b}$ is equal to:

(a) $(a-b)^2$ (b) $(a+b)^2$
(c) $a+b$ (d) $a-b$

8. $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ is equal to:____

(a) a^2+b^2 (b) a^2-b^2
(c) $a-b$ (d) $a+b$

9. The degree of the polynomial $x^2y^2+3xy+y^3$ is ____

(a) 4 (b) 5
(c) 6 (d) 2

10. $x^2-4 =$ _____

(a) $(x-2)(x+2)$ (b) $(x-2)(x-2)$
(c) $(x+2)(x+2)$ (d) None

11. $x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)(........)$

(a) $x^2-1+\frac{1}{x^2}$ (b) $x^2+1+\frac{1}{x^2}$
(c) $x^2+1-\frac{1}{x^2}$ (d) $x^2-1-\frac{1}{x^2}$

12. $2(a^2+b^2) =$ _____

(a) $(a+b)^2+(a-b)^2$
(b) $(a+b)^2-(a-b)^2$
(c) $(a+b)^2$ (d) 4ab

13. Order of surd $\sqrt[3]{x}$ is _____

(a) 3 (b) $\frac{1}{3}$
(c) 0 (d) 1

14. $\dfrac{1}{2-\sqrt{3}}=$ ____

(a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$

(d) $-2+\sqrt{3}$ (d) $-2-\sqrt{3}$

15. $(a+b)^2-(a-b)^2=$ ____

(a) $2(a^2+b^2)$ (b) 4ab

(c) 2ab (d) 3ab

16. $\sqrt{14}.\sqrt{35}=$ ____

(a) $\sqrt[4]{10}$ (b) $\sqrt[5]{10}$

(c) $7\sqrt{10}$ (d) $8\sqrt{10}$

17. A surd which contains a single term is called surd.

(a) Monomial

(b) Binomial

(c) Trinomial

(d) None

ANSWER KEY

1.	a	**2.**	d	**3.**	b	**4.**	a	**5.**	b
6.	b	**7.**	d	**8.**	c	**9.**	a	**10.**	a
11.	a	**12.**	a	**13.**	a	**14.**	a	**15.**	b
16.	c	**17.**	a						